

Como enxergar "a coleção de setas entre X & Y em \mathcal{B} " como funtor?

Notação: Dados categoria \mathcal{B} , objetos X, Y de \mathcal{B} , denotamos por $\mathcal{B}(X, Y)$ ou $\text{Hom}_{\mathcal{B}}(X, Y)$ a coleção de setas de \mathcal{B} com dom X e cod Y .

Variando para baixo do tapete: qual a "natureza" de $\mathcal{C}(X, \cdot)$? Em geral não é conjunto, mas vamos fingir que sim $\ddot{\smile}$.

$\overset{\text{---} X \text{---}}$

- $\mathcal{C}(X, -) : \mathcal{C} \rightarrow \text{Set}$
 $\forall U \in \mathcal{O}(\mathcal{C}) \mapsto \mathcal{C}(X, U)$

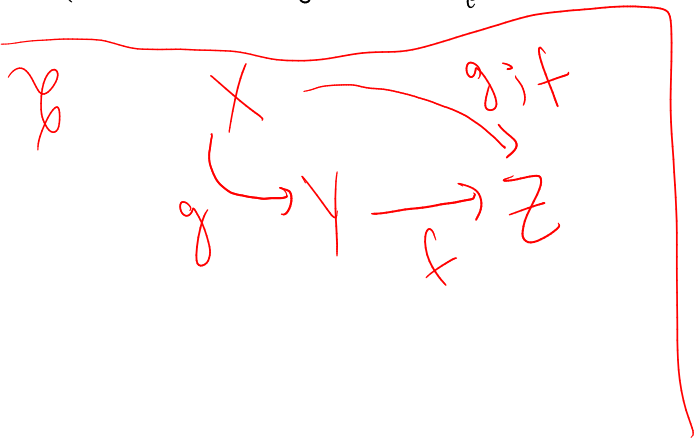
$$f: Y \rightarrow Z \in \mathcal{A}(\mathcal{C}) \mapsto \mathcal{C}(X, f) = \mathcal{C}(X, Y)$$

$$\mathcal{C}(X, f) (g: X \rightarrow Y) = g \circ f$$

$$\downarrow \text{set}$$

$$\mathcal{C}(X, Z)$$

$$\mathcal{C}(X, f) = _ \circ f$$





Preservação de compostas:

A provar: $\mathcal{C}(X, f \circ g) = \underbrace{\mathcal{C}(X, f)}_{\text{Set}} ; \underbrace{\mathcal{C}(X, g)}_{\text{Set}}$

\parallel
 $\underbrace{\mathcal{C}(f \circ g)}_{\text{Set}} = \underbrace{\mathcal{C}(f)}_{\text{Set}} ; \underbrace{\mathcal{C}(g)}_{\text{Set}}$
 $\lambda h. h \circ f \quad ; \quad \lambda k. k \circ g$

$$\left(\overset{\circ}{i} \underset{\circ}{\mathbb{B}} f \right) \underset{\circ}{i} \underset{\circ}{\mathbb{B}} \left(\overset{\circ}{i} \underset{\circ}{\mathbb{B}} g \right) \leq \lambda \mathbb{L} \cdot \left(\overset{\circ}{i} \underset{\circ}{\mathbb{B}} f \right) \overset{\circ}{i} \underset{\circ}{\mathbb{B}} g$$

 =  porque
 $\overset{\circ}{i} \underset{\circ}{\mathbb{B}}$ é associativa!!!

$$= \left(\overset{\circ}{i} \underset{\circ}{\mathbb{B}} f \right) \overset{\circ}{i} \underset{\circ}{\mathbb{B}} g$$

• $\mathcal{C}(-, Y) : \mathcal{C} \rightarrow \text{Set}$

$X \in \text{Ob}(\mathcal{C}) \mapsto \mathcal{C}(X, Y)$

$f : X \xrightarrow{\mathcal{C}} Z \in \text{Ar.}(\mathcal{C}) \mapsto \mathcal{C}(f, Y) : \mathcal{C}(X, Y)$

Em \mathcal{C}

$g \nearrow Y$

$\downarrow \text{set}$
 $\mathcal{C}(Z, Y)$

DEU
RUIM!

$X \xrightarrow{f} Z$

• $\mathcal{C}(-, Y) : \mathcal{C}^{\text{op}} \rightarrow \text{Set}$

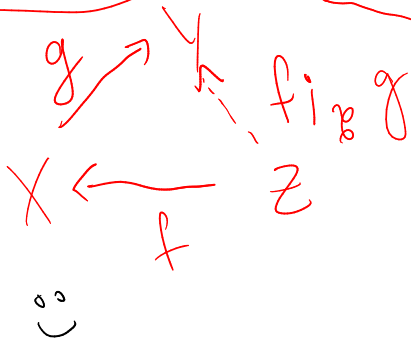
$X \in \text{Ob}(\mathcal{C}) \mapsto \mathcal{C}(X, Y)$

$f: X \rightarrow Z \in \text{Ar}(\mathcal{C}^{\text{op}}) \mapsto \mathcal{C}(f, Y) : \mathcal{C}(X, Y)$

Em \mathcal{C}

DEU

BOM!



\downarrow_{set}
 $\mathcal{C}(Z, Y)$

$\mathcal{C}(f, Y) = f_{i_{\mathcal{C}}}$

$\mathcal{C}(X, -) : \mathcal{C} \rightarrow \text{Set}$, "pos-compar"

$\mathcal{C}(-, 1) : \mathcal{C}^{\text{op}} \rightarrow \text{Set}$, "pre-compar"

$\mathcal{C}(-_0, -_1) : \mathcal{C}^{\text{op}} \times \mathcal{C} \rightarrow \text{Set}$

def v3 : Sejam $\mathcal{C} \begin{matrix} \xrightarrow{F} \\ \xleftarrow{G} \end{matrix} \mathcal{D}$.
Então $F \dashv G$ se

$$\mathcal{C}(_ \circ, G(_ _)) \cong \mathcal{D}(F(_ \circ), _ _)$$

Note que ambos os
functors são

$$\mathcal{C}^{op} \times \mathcal{D} \rightarrow \text{Set}$$

Parece que $\mathcal{L}(-_0, G(-_1))$

$$= (\text{id}(\mathcal{L}^{\mathcal{C}}) \times G); \mathcal{L}(-_0, -_1)$$

e que $\mathcal{L}(F(-_0), -_1)$

$$= (\text{Op} \circ F) \times \text{id}; \mathcal{L}(-_0, -_1)$$

em qual categoria?

Cat!

Na última aula, buscamos $F \dashv X$

No novo formato, queremos $F: \mathcal{C} \rightarrow \mathcal{C} \times \mathcal{C}$

tal que

$$\mathcal{C}(-_0, (-_1 \times -_2)) \cong \mathcal{C} \times \mathcal{C}(F(-_0), (-_1, -_2))$$

Plugando objetos $A, B, C \in \mathcal{U}(\mathcal{C}) \cong \mathcal{U}(\mathcal{C}^{\text{op}})$

$$\mathcal{C}(A, B \times C) \cong \mathcal{C} \times \mathcal{C}(F(A), (B, C))$$

$$\begin{array}{ccc}
 f; \pi_0 & & A \\
 \swarrow & & \downarrow f \\
 B & \leftarrow & B \times C
 \end{array}$$

$$\begin{array}{c}
 \cong \\
 \left(\begin{array}{c}
 \mathcal{C}(F_0(A), B) \\
 \times \\
 \mathcal{C}(F_1(A), C)
 \end{array} \right)
 \end{array}$$

Choose informally: $F(A) = (A, A)$

E nas Setas?

$$f: A \xrightarrow{\text{pop}} B, \quad g: C \xrightarrow{\text{d}} D$$

$$g \times h: C \times C'$$

$$\text{então } F(f): F(B) \rightarrow F(A) \quad h: C' \xrightarrow{\text{d}} D'$$

$$\downarrow \text{pop}$$
$$D \times D'$$

$$\text{Quero: } (B, B) \rightarrow (A, A)$$

$$\mathcal{L}(f, g \times h) \cong \mathcal{L}_{\mathcal{L} \times \mathcal{L}}(F(f), (g, h))$$

$$\left(F_{\text{of}} \underset{\text{oi}}{\rightarrow} g, F_{\text{il}}(f) \underset{\text{ih}}{\rightarrow} h \right) = F(f) \underset{\mathcal{L} \times \mathcal{L}}{\rightarrow} \left(\text{oi} \rightarrow \right) \underset{\mathcal{L} \times \mathcal{L}}{\rightarrow} (g, h)$$

$$f_i \xrightarrow{\varphi_i} (g \times h)$$

=

$$\left\langle \begin{array}{l} f_i \xrightarrow{\varphi_i} \pi_0 \circ g_i \\ f_i \xrightarrow{\varphi_i} \pi_1 \circ h_i \end{array} \right\rangle$$

$$\begin{array}{ccccc} C & \xleftarrow{\pi_0} & C \times C' & \xrightarrow{\pi_1} & C' \\ g \downarrow & & \downarrow \langle \pi_0 \circ g, \pi_1 \circ h \rangle & & \downarrow h \\ D & \xleftarrow{\quad} & D \times D' & \xrightarrow{\quad} & D' \end{array}$$

Querria : $F(f) = (f, f)$

F é o funtor cópia!

Denotado Δ "diagonal"

Quando \mathcal{Y} tem produtos, temos

Quando tem comodutos, $+ \dashv \Delta$

Muitas vezes $1 \times$ tem adj à direita:
Exponencial!

$+ \dashv \Delta \dashv \times \dashv \exp$

Para refletir: repetir com
id no lugar de Δ
quem seriam $+ \& \times ?$