

$f: \mathbb{R} \rightarrow \mathbb{R}$ é contínua se

$$\forall x \in \mathbb{R} \quad \forall \varepsilon > 0 \quad \exists \delta > 0$$

$$\forall y \in \mathbb{R} \quad [|y - x| < \delta \Rightarrow |f(y) - f(x)| < \varepsilon]$$

$f: \mathbb{R} \rightarrow \mathbb{R}$ é unif. contínua se

$$\forall \varepsilon > 0 \quad \exists \delta > 0 \quad \forall x \in \mathbb{R}$$

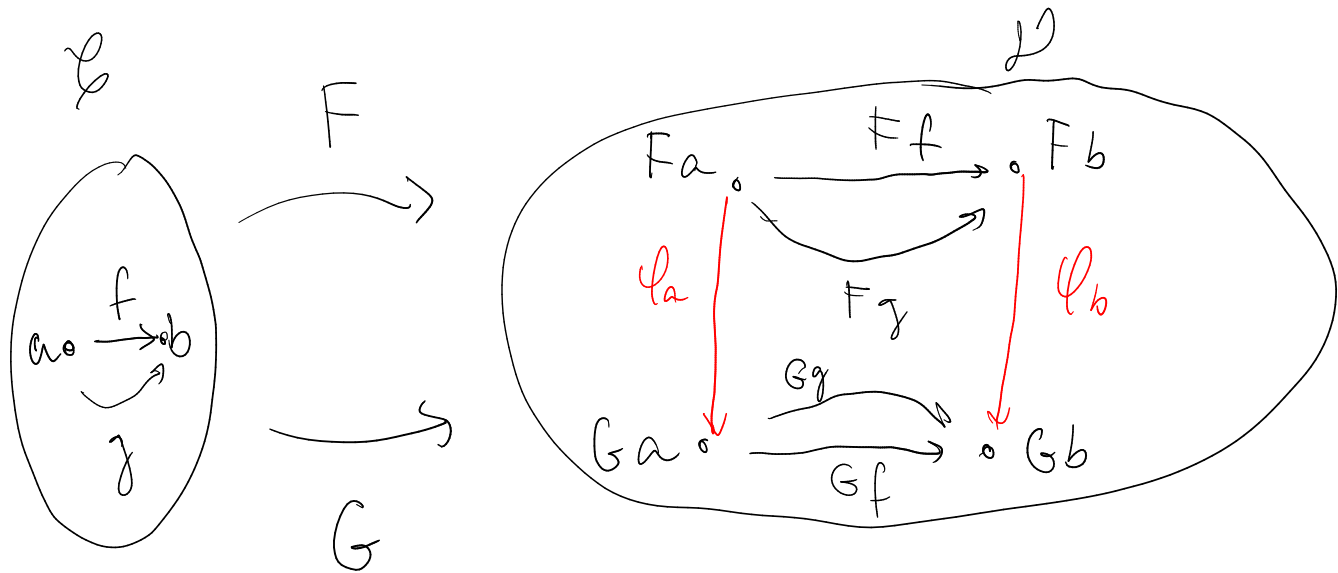
$$\forall y \in \mathbb{R} \quad [|y - x| < \delta \Rightarrow |f(y) - f(x)| < \varepsilon]$$

— x —

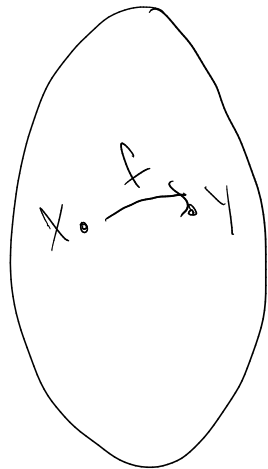
$$f(x, y) = x + y$$

$$g(x, y) = x$$

$$h(x, y) = (y, x, y)$$



Set



Lista



Set

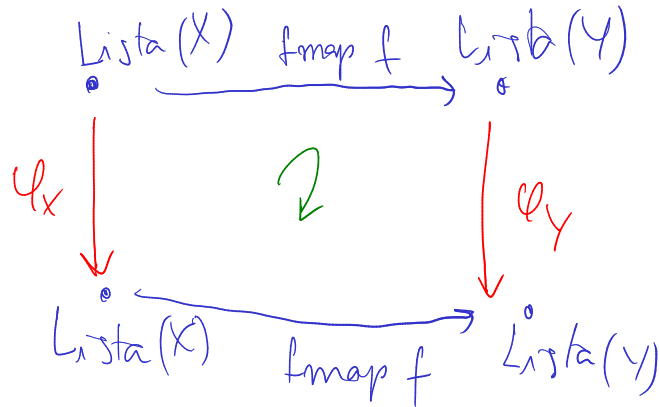


φ : defin : $\forall x \in \mathcal{O}(\text{set})$
 $\varphi_x \in \mathcal{A}(\text{set})$ tal que
 $\varphi_x : \text{Lista}(x) \rightarrow \text{Lista}(x)$

Naturalidade:

$\forall f : x \rightarrow y \in \mathcal{A}(\text{set})$

$(f \text{map } f) ; \varphi_y = \varphi_x ; (f \text{map } f)$



Exemplo não-natural:

φ_N é "soma todos e faz lista unitária com resultado"

φ_X é "reverte a lista"

$f: \mathbb{N} \rightarrow \text{Pal. Ling. Port}$

$n \mapsto \text{nome do } n \text{ em pt-br}$

$[1, 2, 3] \xrightarrow{\text{fmap } f} \{ "um", "dot", "tues" \}$

$\varphi_N \downarrow$

$[6]$

$\xrightarrow{\text{fmap } f}$

\downarrow
 $\{ "tues", "dot", "um" \}$
 \neq
 $["seis"]$

Se tivéssemos ^{um} Monoid em vez de Set ,
a ideia do \mathcal{C}_M acima (mas para todos
os monoides!) funcionaria (exercício)

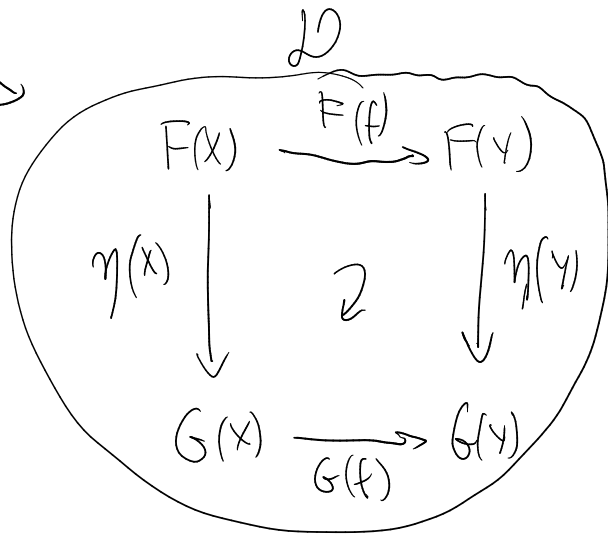
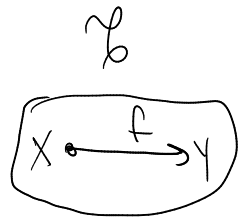
def: Sejam \mathcal{B}, \mathcal{D} categorias,
 $F, G: \mathcal{B} \rightarrow \mathcal{D}$ funtores.

Uma transformação natural η é uma função

$$\eta: \mathcal{U}(\mathcal{B}) \rightarrow \mathcal{A}(\mathcal{D})$$

tal que:

- $\forall X \in \mathcal{U}(\mathcal{B}) \quad (\text{dom}_{\mathcal{D}}(\eta(X)) = F(X) \text{ \& \; } \text{cod}_{\mathcal{D}}(\eta(X)) = G(X))$
- (naturalidade)
- $\forall f \in \mathcal{A}(\mathcal{B}) \quad (F(f) \circ \eta(\text{cod}_{\mathcal{B}}(f)) = \eta(\text{dom}_{\mathcal{B}}(f)) \circ G(f))$



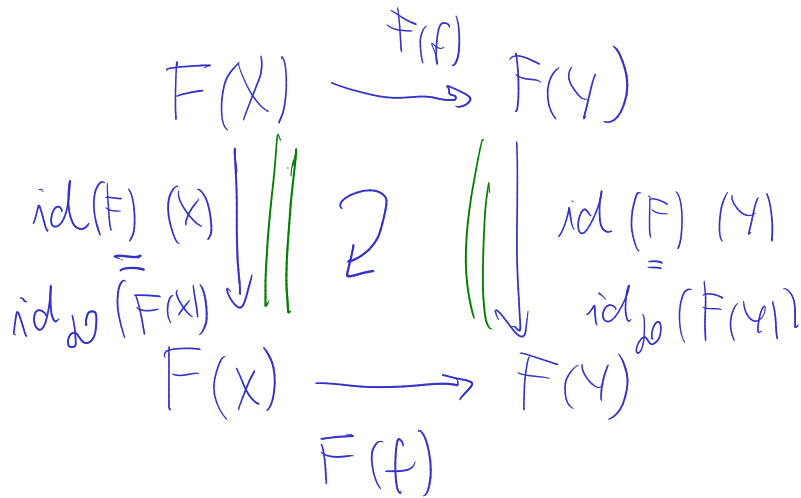
Para que tenhamos categoria (dados \mathcal{B}, \mathcal{D})

- objetos: funtores de \mathcal{B} para \mathcal{D}
- setas: transfs. nats.
- dom: "de qual funtor"
- cod: "para qual funtor"
- $\text{id}(F)$: transf. nat. de F p/ F .

$$\forall x \in \mathcal{O}(\mathcal{B}) \quad \text{id}(F)(x) : F(x) \rightarrow F(x)$$

"id _{\mathcal{D}} (F(x))"

naturalidade :

$$A \text{ f. } X \rightarrow Y \text{ im } \alpha$$


-)

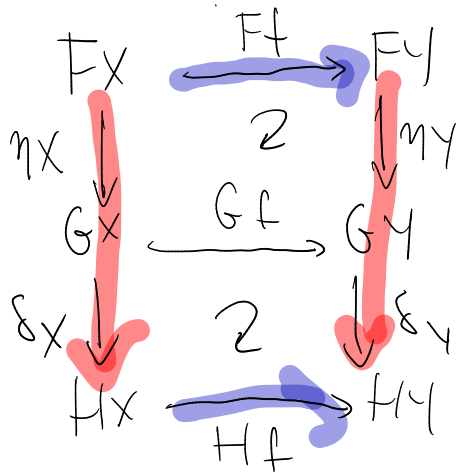
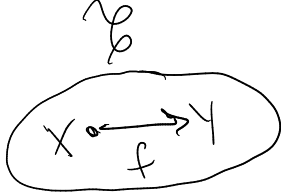
$$\eta: F \rightarrow G$$

$$\delta: G \rightarrow H$$

$$\eta; \delta: F \rightarrow H$$

$$(\eta; \delta)(X): F(X) \rightarrow H(X)$$

$$\eta X; \delta X$$



Tarefa : provar as propriedades
de categorias

Notação $\text{Func}(\mathcal{C}, \mathcal{D})$
 $\mathcal{C}^{\mathcal{D}}$