

Em cada questão abaixo, encontre a forma fechada da função definida por recorrência. Cada questão traz casos de teste para você conferir sua resposta com o auxílio de um computador.

Questão 1.

$$F(n) = \begin{cases} 2, & \text{se } n = 0 \\ 4, & \text{se } n = 1 \\ 8 \cdot F(n-1) - 12 \cdot F(n-2) - 8 \cdot 2^n, & \text{c.c.} \end{cases}$$

$$F(30) = -442147839441333571813376$$

$$F(31) = -2652887036648525416890368$$

$$F(32) = -15917322219892234833100800$$

Questão 2.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 1, & \text{se } n = 1 \\ 9 \cdot F(n-1) - 20 \cdot F(n-2) + 6 \cdot 3^n, & \text{c.c.} \end{cases}$$

$$F(30) = 22294101274601708581723$$

$$F(31) = 111528141330117752146369$$

$$F(32) = 557871257597146730793907$$

Questão 3.

$$F(n) = \begin{cases} 2, & \text{se } n = 0 \\ 2, & \text{se } n = 1 \\ 8, & \text{se } n = 2 \\ 6 \cdot F(n-1) - 12 \cdot F(n-2) + 8 \cdot F(n-3) + 40 \cdot n - 40, & \text{c.c.} \end{cases}$$

$$F(30) = 12554189404808$$

$$F(31) = 26996016937568$$

$$F(32) = 57904749083192$$

$$F(33) = 123909806488080$$

Questão 4.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 1, & \text{se } n = 1 \\ 9 \cdot F(n-1) - 20 \cdot F(n-2) - 4 \cdot 3^n, & \text{c.c.} \end{cases}$$

$$F(30) = -19511660912781152652767$$

$$F(31) = -97604414012009281735511$$

$$F(32) = -488206515264541237971623$$

Questão 5.

$$F(n) = \begin{cases} 16, & \text{se } n = 0 \\ 8, & \text{se } n = 1 \\ 16, & \text{se } n = 2 \\ F(n-1) + 5 \cdot F(n-2) + 3 \cdot F(n-3) - 64 \cdot n^2 - 64 \cdot n, & \text{c.c.} \end{cases}$$

$$\begin{aligned}F(30) &= -6176733962829104 \\F(31) &= -18530201888510872 \\F(32) &= -55590605665543664 \\F(33) &= -166771816996657144\end{aligned}$$

Questão 6.

$$F(n) = \begin{cases} 2, & \text{se } n = 0 \\ 8, & \text{se } n = 1 \\ 16 \cdot F(n-2) + 8 \cdot 4^n, & \text{c.c.} \end{cases}$$

$$\begin{aligned}F(30) &= 140656423562035331072 \\F(31) &= 562625694248141324288 \\F(32) &= 2398076729582241710080\end{aligned}$$

Questão 7.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 1, & \text{se } n = 1 \\ 9 \cdot F(n-1) - 20 \cdot F(n-2) - 10 \cdot 4^n, & \text{c.c.} \end{cases}$$

$$\begin{aligned}F(30) &= -150232994730039258771611 \\F(31) &= -752553091141742937617159 \\F(32) &= -3768502393115638358638371\end{aligned}$$

Questão 8.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 1, & \text{se } n = 1 \\ 7 \cdot F(n-1) - 12 \cdot F(n-2) - 144 \cdot n - 126, & \text{c.c.} \end{cases}$$

$$\begin{aligned}F(30) &= -179805105500172845411 \\F(31) &= -719271071219186662895 \\F(32) &= -2877236232532232500067\end{aligned}$$

Questão 9.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 2, & \text{se } n = 1 \\ 2, & \text{se } n = 2 \\ 15 \cdot F(n-1) - 74 \cdot F(n-2) + 120 \cdot F(n-3) + 2 \cdot 4^n, & \text{c.c.} \end{cases}$$

$$\begin{aligned}F(30) &= 7392385892900636625591690 \\F(31) &= 44477846674590235556664242 \\F(32) &= 267485854967332960997122682 \\F(33) &= 1608013625312345968511102050\end{aligned}$$

Questão 10.

$$F(n) = \begin{cases} 3, & \text{se } n = 0 \\ 6, & \text{se } n = 1 \\ -10 \cdot F(n-1) - 16 \cdot F(n-2) + 81 \cdot n - 45, & \text{c.c.} \end{cases}$$

$$F(30) = 93$$

$$F(31) = 96$$

$$F(32) = 99$$

Questão 11.

$$F(n) = \begin{cases} 6, & \text{se } n = 0 \\ 4, & \text{se } n = 1 \\ 12, & \text{se } n = 2 \\ -F(n-1) + 4 \cdot F(n-2) + 4 \cdot F(n-3) + 108 \cdot n^2 + 108 \cdot n - 96, & \text{c.c.} \end{cases}$$

$$F(30) = 122406547668$$

$$F(31) = 584115530476$$

$$F(32) = 489626248980$$

$$F(33) = 2336462184676$$

Questão 12.

$$F(n) = \begin{cases} 3, & \text{se } n = 0 \\ 6, & \text{se } n = 1 \\ 2 \cdot F(n-1) + 8 \cdot F(n-2) + 162 \cdot n - 36, & \text{c.c.} \end{cases}$$

$$F(30) = 24211351611776171460$$

$$F(31) = 96845406356910374322$$

$$F(32) = 387381625608030125472$$

Questão 13.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 2, & \text{se } n = 1 \\ 2, & \text{se } n = 2 \\ -12 \cdot F(n-1) - 47 \cdot F(n-2) - 60 \cdot F(n-3), & \text{c.c.} \end{cases}$$

$$F(30) = 13000473752787315161522$$

$$F(31) = -65040406937943317971858$$

$$F(32) = 325354195621388842302242$$

$$F(33) = -1627379646540569072640898$$

Questão 14.

$$F(n) = \begin{cases} 7, & \text{se } n = 0 \\ 7, & \text{se } n = 1 \\ -3 \cdot F(n-1) + 10 \cdot F(n-2) - 18 \cdot n - 9, & \text{c.c.} \end{cases}$$

$$F(30) = -3221225372$$

$$F(31) = -6442450841$$

$$F(32) = -12884901782$$

Questão 15.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 1, & \text{se } n = 1 \\ 9 \cdot F(n-1) - 20 \cdot F(n-2) - 8 \cdot 4^n, & \text{c.c.} \end{cases}$$

$$F(30) = -120744266991597008649083$$

$$F(31) = -604832751288426043730279$$

$$F(32) = -3028756995716483897003779$$

Questão 16.

$$F(n) = \begin{cases} 3, & \text{se } n = 0 \\ 3, & \text{se } n = 1 \\ F(n-1) + 2 \cdot F(n-2) + 8 \cdot 3^n, & \text{c.c.} \end{cases}$$

$$F(30) = 3706016755383561$$

$$F(31) = 11118073888470783$$

$$F(32) = 33354268910052633$$

Questão 17.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 2, & \text{se } n = 1 \\ 2, & \text{se } n = 2 \\ -18 \cdot F(n-1) - 107 \cdot F(n-2) - 210 \cdot F(n-3), & \text{c.c.} \end{cases}$$

$$F(30) = 595109275035837775288468262$$

$$F(31) = -4179185241773606078355070420$$

$$F(32) = 29334883784131915539996342206$$

$$F(33) = -205828035002124562146519959788$$

Questão 18.

$$F(n) = \begin{cases} 25, & \text{se } n = 0 \\ 150, & \text{se } n = 1 \\ 150, & \text{se } n = 2 \\ 4 \cdot F(n-1) + 53 \cdot F(n-2) - 168 \cdot F(n-3), & \text{c.c.} \end{cases}$$

$$F(30) = 3578584076117752227868843350$$

$$F(31) = 30657213235075496292365188350$$

$$F(32) = 231057921497381373468087907350$$

$$F(33) = 1947861862660744423085740929150$$

Questão 19.

$$F(n) = \begin{cases} 5, & \text{se } n = 0 \\ 5, & \text{se } n = 1 \\ 9 \cdot F(n-1) - 14 \cdot F(n-2) + 72 \cdot n + 54, & \text{c.c.} \end{cases}$$

$$F(30) = 135236041744153496987572349$$

$$F(31) = 946652292209074736611041773$$

$$F(32) = 6626566045463523671673365429$$

Questão 20.

$$F(n) = \begin{cases} 169, & \text{se } n = 0 \\ 1521, & \text{se } n = 1 \\ 1521, & \text{se } n = 2 \\ 14 \cdot F(n-1) - 9 \cdot F(n-2) - 324 \cdot F(n-3), & \text{c.c.} \end{cases}$$

$$F(30) = -122044144674430460265985366167951$$

$$F(31) = -1138075426214397374354935876218159$$

$$F(32) = -10599781953244314205712087226326799$$

$$F(33) = -98611965634975353384595539644195631$$

Questão 21.

$$F(n) = \begin{cases} 91, & \text{se } n = 0 \\ 546, & \text{se } n = 1 \\ 546, & \text{se } n = 2 \\ -7 \cdot F(n-1) + 26 \cdot F(n-2) + 72 \cdot F(n-3), & \text{c.c.} \end{cases}$$

$$F(30) = -593476215731970091216403132334$$

$$F(31) = 5341285943161468674735974313246$$

$$F(32) = -48071573482158266657470384330254$$

$$F(33) = 432644161364604205577846996928126$$

Questão 22.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 7, & \text{se } n = 1 \\ -14 \cdot F(n-1) - 49 \cdot F(n-2), & \text{c.c.} \end{cases}$$

$$F(30) = -1329821077150843227183931691$$

$$F(31) = 9624298304125594203517607323$$

$$F(32) = -69578943477367000717233849663$$

Questão 23.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 8, & \text{se } n = 1 \\ 16, & \text{se } n = 2 \\ 22 \cdot F(n-1) - 160 \cdot F(n-2) + 384 \cdot F(n-3), & \text{c.c.} \end{cases}$$

$$F(30) = -95324035912010929967527362560$$

$$F(31) = -792297542464863268737208287232$$

$$F(32) = -6576032992617259377074760581120$$

$$F(33) = -54509548833413780405221914050560$$

Questão 24.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 6, & \text{se } n = 1 \\ 6, & \text{se } n = 2 \\ 13 \cdot F(n-1) - 54 \cdot F(n-2) + 72 \cdot F(n-3) + 8 \cdot 3^n, & \text{c.c.} \end{cases}$$

$$F(30) = 1768484302014190894716534$$

$$F(31) = 10611119772464406347790186$$

$$F(32) = 63667574914851593366307414$$

$$F(33) = 382008875969486999933360970$$

Questão 25.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 2, & \text{se } n = 1 \\ 6 \cdot F(n-1) - 8 \cdot F(n-2) - 18 \cdot n + 36, & \text{c.c.} \end{cases}$$

$$F(30) = -1152921493869428924$$

$$F(31) = -4611685996952551618$$

$$F(32) = -18446744030759878856$$

Questão 26.

$$F(n) = \begin{cases} 9, & \text{se } n = 0 \\ 9, & \text{se } n = 1 \\ 9, & \text{se } n = 2 \\ 3 \cdot F(n-2) + 2 \cdot F(n-3) - 84 \cdot n - 60, & \text{c.c.} \end{cases}$$

$$F(30) = -42949671111$$

$$F(31) = -85899346371$$

$$F(32) = -171798689871$$

$$F(33) = -343597384167$$

Questão 27.

$$F(n) = \begin{cases} 234, & \text{se } n = 0 \\ 9828, & \text{se } n = 1 \\ 4914, & \text{se } n = 2 \\ 9828, & \text{se } n = 3 \\ 45 \cdot F(n-2) - 40 \cdot F(n-3) - 84 \cdot F(n-4), & \text{c.c.} \end{cases}$$

$$F(30) = -595318151707088147313912846$$

$$F(31) = 3753376684232439360991144068$$

$$F(32) = -28756739055930282251303964606$$

$$F(33) = 186398559793692454006674645588$$

$$F(34) = -1394181600142764871373955490926$$

Questão 28.

$$F(n) = \begin{cases} 3, & \text{se } n = 0 \\ 3, & \text{se } n = 1 \\ F(n-1) + 2 \cdot F(n-2) + 4 \cdot n + 6, & \text{c.c.} \end{cases}$$

$$F(30) = 8589934527$$

$$F(31) = 17179869111$$

$$F(32) = 34359738299$$

Questão 29.

$$F(n) = \begin{cases} 35, & \text{se } n = 0 \\ 210, & \text{se } n = 1 \\ 210, & \text{se } n = 2 \\ -5 \cdot F(n-1) + 18 \cdot F(n-2) + 72 \cdot F(n-3), & \text{c.c.} \end{cases}$$

$$F(30) = -3094989910092676022618310$$

$$F(31) = 18570389106119586768869970$$

$$F(32) = -111420536097700535820455670$$

$$F(33) = 668530410871982567313419490$$

Questão 30.

$$F(n) = \begin{cases} 1485, & \text{se } n = 0 \\ 5940, & \text{se } n = 1 \\ 11880, & \text{se } n = 2 \\ 11880, & \text{se } n = 3 \\ 7 \cdot F(n-1) + 42 \cdot F(n-2) - 328 \cdot F(n-3) + 448 \cdot F(n-4), & \text{c.c.} \end{cases}$$

$$F(30) = -328078371092953112433647211960$$

$$F(31) = -2605693932238107588573493864440$$

$$F(32) = -20978082816168787932571750426680$$

$$F(33) = -166896943432404037109055563343480$$

$$F(34) = -1341669582781461058249570424746680$$

Questão 31.

$$F(n) = \begin{cases} 1, & \text{se } n = 0 \\ 4, & \text{se } n = 1 \\ 8, & \text{se } n = 2 \\ 14 \cdot F(n-1) - 64 \cdot F(n-2) + 96 \cdot F(n-3) + 2 \cdot 4^n, & \text{c.c.} \end{cases}$$

$$F(30) = 3092771691176723769982976$$

$$F(31) = 18560570832763088822861824$$

$$F(32) = 111380285320661903467347968$$

$$F(33) = 668353691119347035474493440$$

Questão 32.

$$F(n) = \begin{cases} 9, & \text{se } n = 0 \\ 9, & \text{se } n = 1 \\ F(n-1) + 20 \cdot F(n-2) - 9 \cdot 4^n, & \text{c.c.} \end{cases}$$

$$F(30) = -10221489890678126732355$$

$$F(31) = -51148954627556480152911$$

$$F(32) = -255744773137782400764555$$

Questão 33.

$$F(n) = \begin{cases} 8, & \text{se } n = 0 \\ 6, & \text{se } n = 1 \\ 72, & \text{se } n = 2 \\ 9 \cdot F(n-1) + 9 \cdot F(n-2) - 81 \cdot F(n-3), & \text{c.c.} \end{cases}$$

$$F(30) = 1647129056757192$$

$$F(31) = 1235346792567894$$

$$F(32) = 14824161510814728$$

$$F(33) = 11118121133111046$$

Questão 34.

$$F(n) = \begin{cases} 2, & \text{se } n = 0 \\ 4, & \text{se } n = 1 \\ 8 \cdot F(n-1) - 12 \cdot F(n-2), & \text{c.c.} \end{cases}$$

$$F(30) = 2147483648$$

$$F(31) = 4294967296$$

$$F(32) = 8589934592$$

Questão 35.

$$F(n) = \begin{cases} 9, & \text{se } n = 0 \\ 9, & \text{se } n = 1 \\ 9, & \text{se } n = 2 \\ 3 \cdot F(n-2) + 2 \cdot F(n-3) - 108 \cdot n^2 - 108 \cdot n - 90, & \text{c.c.} \end{cases}$$

$$F(30) = -289910257371$$

$$F(31) = -579820557681$$

$$F(32) = -1159641130761$$

$$F(33) = -2319282309051$$

Polinômios e raízes

$x^2 - 9 \cdot x + 14$	2	7		
$x^2 - 9 \cdot x + 20$	4	5		
$x^2 - 8 \cdot x + 12$	2	6		
$x^2 - 7 \cdot x + 12$	3	4		
$x^2 - 6 \cdot x + 8$	2	4		
$x^2 - 2 \cdot x - 8$	-2	4		
$x^2 - x - 20$	-4	5		
$x^2 - x - 2$	-1	2		
$x^2 - 16$	-4	4		
$x^2 + 3 \cdot x - 10$	-5	2		
$x^2 + 10 \cdot x + 16$	-8	-2		
$x^2 + 14 \cdot x + 49$	-7	-7		
$x^3 - 22 \cdot x^2 + 160 \cdot x - 384$	6	8	8	
$x^3 - 15 \cdot x^2 + 74 \cdot x - 120$	4	5	6	
$x^3 - 14 \cdot x^2 + 9 \cdot x + 324$	-4	9	9	
$x^3 - 14 \cdot x^2 + 64 \cdot x - 96$	4	4	6	
$x^3 - 13 \cdot x^2 + 54 \cdot x - 72$	3	4	6	
$x^3 - 9 \cdot x^2 - 9 \cdot x + 81$	-3	3	9	
$x^3 - 6 \cdot x^2 + 12 \cdot x - 8$	2	2	2	
$x^3 - 4 \cdot x^2 - 53 \cdot x + 168$	-7	3	8	
$x^3 - x^2 - 5 \cdot x - 3$	-1	-1	3	
$x^3 - 3 \cdot x - 2$	-1	-1	2	
$x^3 + x^2 - 4 \cdot x - 4$	-2	-1	2	
$x^3 + 5 \cdot x^2 - 18 \cdot x - 72$	-6	-3	4	
$x^3 + 7 \cdot x^2 - 26 \cdot x - 72$	-9	-2	4	
$x^3 + 12 \cdot x^2 + 47 \cdot x + 60$	-5	-4	-3	
$x^3 + 18 \cdot x^2 + 107 \cdot x + 210$	-7	-6	-5	
$x^4 - 7 \cdot x^3 - 42 \cdot x^2 + 328 \cdot x - 448$	-7	2	4	8
$x^4 - 45 \cdot x^2 + 40 \cdot x + 84$	-7	-1	2	6